



When D-branes break

Oren Bergman^a, Gilad Lifschytz^{b,*}

^a *Department of Physics, Technion, Israel Institute of Technology, Haifa 32000, Israel*

^b *Department of Mathematics and Physics and CCMSC, University of Haifa at Oranim, Tivon 36006, Israel*

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Abstract

We analyze the possible configurations of D-branes breaking on other D-branes. We describe these configurations in the context of a brane–antibrane effective theory in two ways. First as a tachyon configuration representing a non-trivial bundle over the sphere surrounding the end of the brane à la Polchinski, and second in terms of tachyon solitons using homotopy theory. Surprisingly, in some cases there are topologically stable configurations of broken branes.

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1. Introduction

In string theory there are several circumstances in which branes can terminate on other branes. Strings can of course end on D-branes, and using S- and T-duality one can show that a Dp -brane can end on a $D(p+2)$ -brane, and that a Dp -brane with $p \leq 5$ can end on an NS5-brane. These configurations were shown to be consistent with charge conservation by Strominger [1]. For a p -brane to end on a q -brane requires a coupling between the $(p+1)$ -form gauge field and the worldvolume gauge field on the q -brane. This in turn implies that the end of the p -brane is charged under the worldvolume gauge field on the q -brane. For example, a string can end on a Dp -brane because there is a term $*F \wedge B$ in the expansion of the DBI action on the Dp -brane. This coupling implies that the end of the string is charged electrically under the worldvolume gauge field. Similarly, a Dp -brane can end on a $D(p+2)$ -brane because there is a term $F \wedge C_{p+1}$ in CS action on the $D(p+2)$ -brane. This gives the $(p-1)$ -dimensional end of the p -brane a magnetic charge under the worldvolume gauge field in the $D(p+2)$ -brane. The full CS action on a Dq -brane

contains more terms coupling the worldvolume gauge field to all lower rank RR fields,

$$S_{CS} = C_{q+1} + C_{q-1} \wedge F + \frac{1}{2} C_{q-3} \wedge F \wedge F + \dots, \quad (1.1)$$

which suggests that any Dp -brane can end on a Dq -brane with $p < q$, as long as the end of the p -brane carries a charge of the form

$$\int_{S^{q-p}} \text{Tr } F \wedge F \wedge \dots \wedge F. \quad (1.2)$$

These possibilities were mentioned briefly in [2]. However this requires a large enough gauge group such that the gauge bundle with the above Chern class is non-trivial, so we need several coincident q -branes for the p -brane to end.

Charge conservation is a necessary condition for these configurations to exist, but it is not obviously sufficient. Even if they do exist, they may not be stable.¹ Polchinski has recently provided an explicit construction for the case of a 1-brane ending on 9-branes in type I string theory, in terms of an effective tachyon field theory on a system of 9-branes and anti-9-branes

* Corresponding author.

E-mail addresses: bergman@physics.technion.ac.il (O. Bergman), giladl@research.haifa.ac.il (G. Lifschytz).

¹ Existence and stability are separate questions. For example, neutral non-BPS Dp -branes of the “wrong” dimension p exist in type II string theory (p odd in type IIA and even in type IIB), but they are not stable. On the other hand neutral Dp -branes with the same p as the charged BPS D-branes do not exist.

[3].² The basic ingredient is Sen's construction, and Witten's generalization, of Dp-branes as topological solitons in a higher-dimensional brane–antibrane system [5,6]. In particular the 1-brane in type I string theory corresponds to a co-dimension eight tachyon configuration in the effective field theory of eight 9-brane–anti-9-brane pairs, which asymptotically behaves as $T(x) \sim \Gamma \cdot x$. Adding eight more 9-branes, Polchinski showed that the new system admits a tachyon configuration that describes a semi-infinite 1-brane, namely a configuration that reduces to the above behavior on one side of the coordinate axis, and to the vacuum on the other side.³ The obvious interpretation is that the 1-brane ends on the eight 9-branes that remain after the tachyon condenses. This agrees with the requirement (1.2), since $O(8)$ admits a non-trivial bundle over S^8 .

We will generalize this construction to all the Dp-branes of type I and type IIB string theory, and show that all the (stable) p-branes can end on 9-branes, provided there are enough 9-branes. This will be in accord with the charge conservation condition (1.2). We will also show that the existence of a configuration where a p-brane ends on 9-branes is connected to the existence of a $(p-1)$ -dimensional soliton in a Higgs phase of the 9-brane–anti-9-brane model, where the extra 9-branes are separated (by Wilson lines) from the brane–antibrane pairs. This soliton sits at the boundary of the p-brane in the 9-brane, and carries the charge of (1.2). Interestingly we will find examples of p-branes with both ends on 9-branes which are topologically stable.

In Section 2 we will review Polchinski's construction, and generalize it to p-branes ending on 9-branes in type I and type IIB string theory. In Section 3 we will analyze the Higgs phase of the brane–antibrane model, and describe the end of the p-brane as a $(p-1)$ -dimensional topological soliton. Section 4 contains our discussion.

2. Tachyon model

2.1. Open type I D1-brane

In [3] Polchinski constructed a tachyon configuration which describes a semi-infinite D1-brane in type I string theory. He considered a sub-system of 16 9-branes and 8 anti-9-branes, with a gauge group $O(16) \times O(8)$. Since the tachyon is a bi-fundamental field it can be thought of as a map from R^8 to R^{16} . The vev of the tachyon, up to a multiplicative constant, is given by

$$T_0 = \begin{bmatrix} I_{8 \times 8} \\ 0_{8 \times 8} \end{bmatrix} \text{ mod } O(16), \quad (2.1)$$

so it reduces to a map between the spheres

$$T_0: S^7 \rightarrow S^{15}. \quad (2.2)$$

² In [4] Hirano and Hashimoto constructed a D8-brane with a D6-brane ending on it as a non-uniform 8-brane soliton in unstable D9-branes in type IIA string theory.

³ Other aspects of asymmetric brane–antibrane systems were studied in [7,8].

Polchinski then considers a tachyon configuration which asymptotically on an S^8 surrounding the origin is given by the third Hopf map

$$T(\Omega): S^7 \rightarrow S^{15} \rightarrow S^8. \quad (2.3)$$

The configuration is explicitly

$$T(\Omega) = \frac{r + \sum_{i=1}^9 \gamma^i x^i}{\sqrt{2r(r+x^9)}} T_0, \quad (2.4)$$

where γ^i are the $SO(9)$ gamma matrices. Near the positive x^9 axis

$$T \sim \begin{bmatrix} I \\ 0 \end{bmatrix} = T_0, \quad (2.5)$$

which is the vacuum, and near the negative x^9 axis

$$T \sim \begin{bmatrix} 0 \\ \sum_{i=1}^8 \Gamma^i x^i / |x| \end{bmatrix}, \quad (2.6)$$

where Γ^i are the $SO(8)$ gamma matrices. This is precisely the asymptotic form of the tachyon for the D1-brane. The configuration (2.4) therefore describes a semi-infinite D1-brane, with an endpoint at $x^9 = 0$.

The tachyon configuration is accompanied by a gauge field configuration in the unbroken $O(8)$ gauge group such that

$$\int_{S^8} \text{Tr } F^4 = 1. \quad (2.7)$$

Note however that the gauge bundle is classified by $\pi_7(O(8)) = \mathbb{Z} \oplus \mathbb{Z}$. There seem to be two inequivalent charges that the endpoint of the 1-brane can carry. We will come back to this point and its implication at the end of Section 3.

2.2. Other branes

Let us try to extend Polchinski's construction to the other D-branes in type I string theory. The branes which might be able to end on 9-branes are D1, D5, D7 and D8. Interestingly there are exactly four Hopf maps,

$$\begin{aligned} \text{0th: } & S^0 \rightarrow S^1 \rightarrow S^1, \\ \text{1st: } & S^1 \rightarrow S^3 \rightarrow S^2, \\ \text{2nd: } & S^3 \rightarrow S^7 \rightarrow S^4, \\ \text{3rd: } & S^7 \rightarrow S^{15} \rightarrow S^8, \end{aligned} \quad (2.8)$$

which naturally correspond to an open 8-brane, 7-brane, 5-brane and 1-brane, respectively. The construction is a simple generalization of the 1-brane case. We begin with $2k$ 9-branes and k anti-9-branes, i.e. $O(2k) \times O(k)$. The tachyon is therefore a map from R^k to R^{2k} , and its vev reduces to a map from S^{k-1} to S^{2k-1} . For $k = 1, 2, 4$ and 8 , there exists a non-trivial bundle

$$S^{k-1} \rightarrow S^{2k-1} \rightarrow S^k, \quad (2.9)$$

which can be used to construct the asymptotic form of the tachyon for an open $(9 - k)$ -brane,

$$T(\Omega) = \frac{r + \sum_{i=1}^{k+1} \gamma^i x^i}{\sqrt{2r(r + x^{k+1})}} T_0. \quad (2.10)$$

However this raises a couple of puzzles. The topological class of this bundle takes values in $\pi_{k-1}(O(k))$, where the $k - 1$ refers to the base S^k and the $O(k)$ refers to the fiber S^{k-1} . In the four cases above,

$$\begin{aligned} \pi_0(O(1)) &= \mathbb{Z}_2, & \pi_1(O(2)) &= \mathbb{Z}, \\ \pi_3(O(4)) &= \mathbb{Z} \oplus \mathbb{Z}, & \pi_7(O(8)) &= \mathbb{Z} \oplus \mathbb{Z}. \end{aligned} \quad (2.11)$$

The problem is that, other than the $k = 1$ case, these homotopy groups do not agree with the corresponding D-brane charges.

In fact the same problem shows up in the construction of the infinite D p -branes as tachyonic solitons in a system with k 9-brane–anti-9-brane pairs. The resulting charges take values in precisely the groups shown in (2.11). In that case we know that in order to get reliable answers we need to take a larger number N of brane–antibrane pairs, so that the homotopy groups are stable. This follows from the correct identification of D-brane charge in K-theory [6].

We propose that the same is true here. With $N + k$ 9-branes and N anti-9-branes the tachyon vev defines a map

$$S^{N-1} \rightarrow S^{N+k-1}. \quad (2.12)$$

In general S^{N+k-1} cannot be obtained by fibring S^{N-1} over S^k . This is only true for the four Hopf maps (2.8). But a more general class of sphere bundles exists

$$S^{N-1} \rightarrow M^{N+k-1} \rightarrow S^k, \quad (2.13)$$

where the total space M^{N+k-1} is not in general a sphere. These bundles are classified by $\pi_{k-1}(O(N))$, and therefore agree precisely, for large N , with the D-brane charges. The non-triviality of the bundle shows that a p -brane can only end on precisely $k = 9 - p$ 9-branes.

We do not have an explicit expression for the asymptotic form of the tachyon in this more general case, but we can suggest a possible way to construct the map (2.12). Start with the inclusion map of the bundle $S^{N-1} \rightarrow M^{N-k-1}$, and then construct a map $M^{N-k-1} \rightarrow S^{N-k-1}$ by mapping the neighborhood of a point in M^{N-k-1} to $S^{N-k-1} - \{0\}$, and the complement of the neighborhood to $\{0\}$. The tachyon is the composite map of these two maps.⁴

2.3. Type IIB

In type IIB string theory we expect to see 1-branes, 3-branes, 5-branes and 7-branes ending on 9-branes. Starting again with $2k$ 9-branes and k anti-9-branes, the gauge group is $U(2k) \times U(k)$, and the tachyon vev defines a map from S^{2k-1} to S^{4k-1} . There are now three Hopf maps

$$S^{2k-1} \rightarrow S^{4k-1} \rightarrow S^{2k}, \quad (2.14)$$

with $k = 1, 2$ and 4. This gives correspondingly an open 7-brane, 5-brane and 1-brane. The charges come out right in these cases since the groups $\pi_{2k-1}(U(k)) = \mathbb{Z}$ are stable, and we do not need to add more 9-brane–anti-9-brane pairs. On the other hand there is no Hopf map for the 3-brane. We are therefore forced to consider the more general class of unitary sphere bundles. In particular, the open 3-brane can be constructed as

$$S^5 \rightarrow M^{11} \rightarrow S^6. \quad (2.15)$$

More generally, a configuration of $N + k$ 9-branes and N anti-9-branes will give rise to an open $(9 - 2k)$ -brane as the bundle

$$S^{2N-1} \rightarrow M^{2(N+k)-1} \rightarrow S^{2k}. \quad (2.16)$$

These bundles are classified by $\pi_{2k-1}(U(N)) = \mathbb{Z}$, which agrees with the D-brane charges.

3. Higgs–tachyon model

Let us now go back to the description of D-branes as topological solitons in a system with an equal number of 9-branes and anti-9-branes, and consider the effect of adding extra 9-branes. For N brane–antibrane pairs the effective field theory has a gauge group $G_N \times G_N$, where G_N is either $O(N)$, $U(N)$ or $Sp(N)$, depending on the theory, and a bi-fundamental tachyon field. The lower dimensional p -branes are classified by the homotopy groups of the tachyon vacuum manifold $\pi_{8-p}((G_N \times G_N)/G_N) = \pi_{8-p}(G_N)$. For $U(N)$ this gives all the odd p BPS D-branes of type IIB, and for $O(N)$ it gives the BPS 1-brane and 5-brane, and the stable non-BPS 0-brane, 7-brane, and 8-brane. The $Sp(N)$ case corresponds to the other orientifold projection of type IIB [9], in which case we get a BPS 1-brane and 5-brane, and a stable non-BPS 3-brane and 4-brane.

With k extra 9-branes the gauge group is $G_{N+k} \times G_N$, and the subgroup which is left unbroken by the tachyon is $G_k \times G_N$. The resulting vacuum manifold G_{N+k}/G_k satisfies (see Appendix A)

$$\pi_{8-p}(G_{N+k}/G_k) = 0 \quad (3.1)$$

for $p > 8 - d$, where $d = k, 2k + 1$ and $4k + 3$ in the O , U and Sp cases, respectively. It therefore appears that the p -branes become topologically unstable one-by-one as we increase k . For each p -brane there is a critical number of excess 9-branes k at which it loses its topological charge. For example, the soliton describing the D1-brane in type I string theory becomes topologically unstable when $k = 8$.⁵ This is precisely the number of extra 9-branes that were needed for the construction of the semi-infinite 1-brane in the previous section. In fact the smallest value of k for any p -brane soliton to become topologically unstable is the same as that needed for the construction of the semi-infinite p -brane. This suggests that the p -brane becomes topologically unstable as a soliton because it can break open on the 9-branes. As we will now show, this is related to the existence of a $(p - 1)$ -dimensional soliton.

⁴ We thank Edward Witten for suggesting this to us.

⁵ Of course the tadpole condition fixes $k = 32$ in type I so none of the solitons are topologically stable [10], but the statement is still correct.

3.1. A simple field theory example

Consider a four-dimensional $SU(2)$ gauge theory with two scalar fields, a triplet and a doublet. Assume that both have a non-zero vev, and that the triplet vev is larger than the doublet vev. The gauge symmetry is broken in two stages:

$$SU(2) \rightarrow U(1) \rightarrow 0. \quad (3.2)$$

The second stage by itself is the Abelian Higgs model, which admits a Nielsen–Olesen string soliton with charge in $\pi_1(U(1)) = \mathbb{Z}$. This string confines the magnetic flux of the broken $U(1)$. The first stage is the 't Hooft–Polyakov model, which gives rise to a magnetic monopole with charge in $\pi_2(SU(2)/U(1)) = \mathbb{Z}$. The full theory has no topologically stable solitons since $\pi_1(SU(2)) = \pi_2(SU(2)) = 0$. The magnetic flux of the monopole from stage 1 gets confined into a semi-infinite string in stage 2. Alternatively, the string from stage 2 can break by the creation of a monopole–antimonopole pair in stage 1. Either way we see that the loss of topological charge is explained by the configuration of a string ending on a monopole.

3.2. Type II

The simple field theory example above illustrates our strategy for the brane–antibrane system. We first separate the extra branes from the brane–antibrane pairs by turning on a vev for the appropriate adjoint scalars, and then let the tachyon condense.⁶ We will call the first stage the “Higgs stage”, and the second the “tachyon stage”. In the type IIB 9-brane–anti-9-brane system this gives

$$\begin{aligned} U(N+k) \times U(N) \\ \rightarrow U(k) \times U(N) \times U(N) \rightarrow U(k) \times U(N). \end{aligned} \quad (3.3)$$

The vacuum manifold of the full theory is given by

$$\mathcal{M}_k = \frac{U(N+k)}{U(k)}, \quad (3.4)$$

which has trivial homotopy groups $\pi_{8-p}(\mathcal{M}_k) = 0$ for $p \geq 8 - 2k$. The vacuum manifolds of the Higgs and tachyon stage are given by

$$\mathcal{M}_k^H = \frac{U(N+k)}{U(k) \times U(N)}, \quad (3.5)$$

$$\mathcal{M}_k^T = U(N). \quad (3.6)$$

The homotopy groups of the three manifolds above have different physical meanings. We already saw that $\pi_{8-p}(\mathcal{M}_k)$ classifies p -dimensional solitons in the full theory. The group $\pi_{8-p}(\mathcal{M}_k^T)$ classifies p -dimensional solitons in the tachyon stage, which is the brane–antibrane annihilation stage. These are precisely the p -branes. Finally, the group $\pi_{9-p}(\mathcal{M}_k^H)$ classifies $(p-1)$ -dimensional solitons in the Higgs stage. In the

simple example above we saw that the existence of a pointlike soliton in the first stage is what allowed the string-like soliton of the second stage to break. We shall therefore refer to the solitons in the Higgs stage as *end-solitons*.⁷ In general a p -brane will be able to break if there exists at least one matching $(p-1)$ -dimensional end-soliton in the Higgs stage.

The homotopy groups of the three manifolds form an exact sequence given by

$$\begin{aligned} \cdots \rightarrow \pi_{9-p}(\mathcal{M}_k) \xrightarrow{j^*} \pi_{9-p}(\mathcal{M}_k^H) \xrightarrow{\partial^*} \pi_{8-p}(\mathcal{M}_k^T) \\ \xrightarrow{i^*} \pi_{8-p}(\mathcal{M}_k) \rightarrow \cdots \end{aligned} \quad (3.7)$$

This implies that when $\pi_{8-p}(\mathcal{M}_k) = 0$ the map

$$\pi_{9-p}(\mathcal{M}_k^H) \xrightarrow{\partial^*} \pi_{8-p}(\mathcal{M}_k^T) \quad (3.8)$$

is onto (surjective), because $\text{Im } \partial^* = \text{Ker } i^* = \pi_{8-p}(\mathcal{M}_k^T)$. In other words there is at least one end-soliton for each p -brane with $p \geq 8 - 2k$. Therefore the p -brane can break precisely when it is no longer stable as a soliton in the full theory, which is what we wanted to demonstrate.

Since there are only odd-dimensional stable p -branes in type IIB to begin with, the smallest p -brane that can break has $p = 9 - 2k$. For $p \geq 9 - 2k$ the exact sequence actually shows that the map (3.8) is an isomorphism, since in this case both $\pi_{8-p}(\mathcal{M}_k) = \pi_{9-p}(\mathcal{M}_k) = 0$. This means there is only one possible end-soliton for each p -brane. This will not be the case in type I.

The $(p-1)$ -dimensional end-soliton has the correct charge in the unbroken gauge group $U(k)$ to satisfy the condition of section 2,

$$\int_{S^{9-p}} \text{Tr } F^{(9-p)/2} \in \pi_{8-p}(U(k)) = \mathbb{Z} \quad \text{for } p \geq 9 - 2k. \quad (3.9)$$

As a consistency check we note that the gauge charge is identical to the topological charge of the end-soliton in $\pi_{9-p}(\mathcal{M}_k^H)$ thanks to identity (A.12). Fig. 1 shows a 7-brane breaking on a 9-brane (or a 1-brane on a 3-brane).

3.3. Type I

Applying the same strategy in type I string theory, we break the 9-brane–anti-9-brane gauge symmetry in two stages by first separating the extra 9-branes (turning on the appropriate Wilson line), and then letting the tachyon condense,

$$\begin{aligned} O(N+k) \times O(N) \\ \rightarrow O(k) \times O(N) \times O(N) \rightarrow O(k) \times O(N). \end{aligned} \quad (3.10)$$

The three vacuum manifolds in this case are given by

$$\mathcal{M}_k = \frac{O(N+k)}{O(k)}, \quad (3.11)$$

⁶ Since we are using 9-branes we cannot really separate them, but we can compactify them and turn on Wilson lines. This is T-dual to separating lower-dimensional branes.

⁷ These are solitons in the topological sense. Whether they have a finite energy or not should be determined by solving the equations of motion of the particular model.

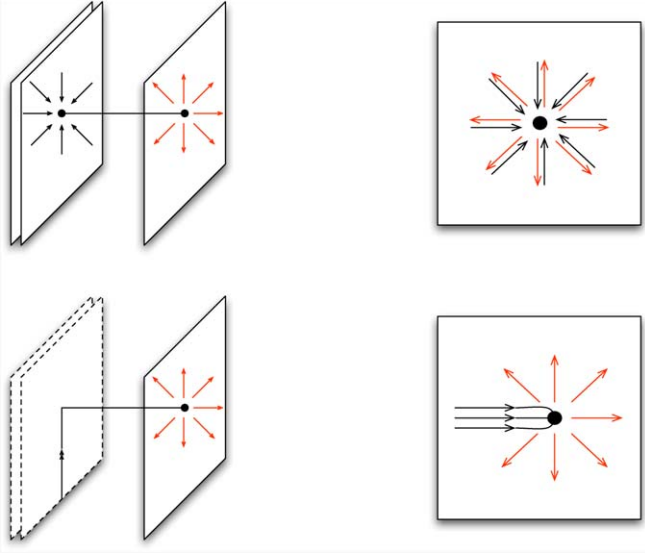


Fig. 1. A p -brane breaking on $(p+2)$ -branes from the spacetime and world-volume points of view. On top an 't Hooft–Polyakov monopole forms when we separate the brane–antibrane pair from the extra brane. On the bottom the magnetic flux on the brane–antibrane is confined to a semi-infinite p -brane after the tachyon condenses.

$$\mathcal{M}_k^H = \frac{O(N+k)}{O(k) \times O(N)}, \quad (3.12)$$

$$\mathcal{M}_k^T = O(N). \quad (3.13)$$

The solitons of the full theory are classified by $\pi_{8-p}(\mathcal{M}_k)$, which in this case vanishes for $p \geq 9 - k$. We therefore expect the 8-brane to first break when $k = 1$, the 7-brane when $k = 2$, the 5-brane when $k = 4$, and the 1-brane when $k = 8$. This is confirmed, exactly as in the type IIB case, by looking at the exact homotopy sequence (3.7). When $\pi_{8-p}(\mathcal{M}_k) = 0$, the map $\pi_{9-p}(\mathcal{M}_k^H) \rightarrow \pi_{8-p}(\mathcal{M}_k^T)$ is onto, and therefore each p -brane with $p \geq 9 - k$ has at least one end-soliton associated to it.

For $p \geq 10 - k$ the map is an isomorphism, so there is exactly one end-soliton for each p -brane. However now the smallest p -brane that can break has $p = 9 - k$, for which the map is onto, but not one-to-one. The end-solitons in these cases are not in one-to-one correspondence with the p -branes.

This can have an interesting effect. Consider a finite open p -brane with two ends. If the group of end-solitons is isomorphic to the group of p -branes, the two ends carry opposite charges and the configuration is unstable. However if the two groups are not isomorphic, it should be possible to have an open p -brane with ends whose charges do not sum up to zero. Let us look at each of the four p -branes in turn.

1-brane: For the 1-brane with $k = 8$ the particle-like end-solitons are classified by

$$\pi_8(\mathcal{M}_8^H) = \mathbb{Z} \oplus \mathbb{Z}, \quad (3.14)$$

whereas the 1-brane itself is in $\pi_7(\mathcal{M}_8^T) = \mathbb{Z}$. There are two inequivalent types of end-solitons. Therefore an open 1-brane with one end on one type of soliton, and the other end on the other type of (anti-) soliton should be stable. Indeed, the full

theory contains a stable particle-like soliton in this case, since

$$\pi_8(\mathcal{M}_8) = \mathbb{Z}. \quad (3.15)$$

5-brane: The 5-brane with $k = 4$ can likewise break in two inequivalent ways, since $\pi_4(\mathcal{M}_4^H) = \mathbb{Z} \oplus \mathbb{Z}$. This leads to a stable 4-dimensional soliton, as can be seen from $\pi_4(\mathcal{M}_4) = \mathbb{Z}$.

7-brane: For the 7-brane with $k = 2$ the possible end-solitons are classified by

$$\pi_2(\mathcal{M}_2^H) = \mathbb{Z}, \quad (3.16)$$

whereas the 7-brane is in $\pi_1(\mathcal{M}_2^T) = \mathbb{Z}_2$. In this case there is a single type of end-soliton, but since the 7-brane does not have an orientation (it is its own antibrane) the two endpoints can be charged equally or oppositely. In the former case the object should be stable. This is confirmed by the existence of a stable 6-dimensional soliton in

$$\pi_2(\mathcal{M}_2) = \mathbb{Z}. \quad (3.17)$$

Furthermore, from the part of the exact sequence

$$\begin{array}{ccccc} \pi_2(\mathcal{M}_2) & \rightarrow & \pi_2(\mathcal{M}_2^H) & \rightarrow & \pi_1(\mathcal{M}_2^T) \\ \mathbb{Z} & \rightarrow & \mathbb{Z} & \rightarrow & \mathbb{Z}_2 \end{array} \quad (3.18)$$

we see that the basic soliton carries two units of end-soliton charge.

8-brane: In the case of the 8-brane with $k = 1$ the end-solitons carry a \mathbb{Z}_2 charge, and are therefore in one-to-one correspondence with the 8-brane. Correspondingly, there is no stable 7-dimensional soliton in the full theory.

Finally, as in the type II case, the end-solitons also carry the required charges in the unbroken $O(k)$

$$\int_{S^{9-p}} \text{Tr } F^{(9-p)/2} \in \pi_{8-p}(O(k)), \quad (3.19)$$

which agree with their topological charge in $\pi_{9-p}(\mathcal{M}_k^H)$ thanks to (A.12).⁸

4. Discussion

We have given two descriptions of p -branes ending on 9-branes. The two descriptions agree on the conditions for branes to end, but they are valid in different regimes. In the tachyon model all the 9-branes and anti-9-branes coincide (no Wilson line), so the broken p -brane is inside the branes it ends on. In the Higgs–tachyon model we first break the symmetry by separating the extra 9-branes from the brane–antibrane pairs, and then let the tachyon condense. The p -brane forms at the location of the pairs, and breaks by connecting to a p -brane which stretches between the pairs and the extra 9-branes (Fig. 1). The former is the p -dimensional soliton formed in the

⁸ We are assuming here that there is a proper generalization of the RR charge conservation argument which led to (1.2) for the case of the type I 7-brane and 8-brane, whose RR fields exist only as torsion in K-theory. It would be interesting to formulate this in a precise manner.

tachyon stage, and the latter is the $(p - 1)$ -dimensional end-soliton formed in the Higgs stage.

An interesting question is whether we can connect the two descriptions by gradually reducing the Higgs vev (or really the Wilson line in the 9-brane case), i.e. the separation between the brane–antibrane pairs and the extra 9-branes. As we reduce the Higgs vev the core of the end-soliton grows, until it fills all of space, at which point the full gauge group $G_{N+k} \times G_N$ would be unbroken everywhere. This clearly does not fit with the first description, in which the gauge group is broken to $G_k \times G_N$ almost everywhere. Furthermore, the semi-infinite part of the p -brane carries different fluxes in the two descriptions. In the Higgs–tachyon model the p -brane carries the magnetic flux of the gauge group which is broken in the tachyon stage, whereas in the tachyon model it carries the unbroken flux. Apparently a phase transition must occur between the two descriptions.

We found three examples of topologically stable open p -branes in 9-branes with orthogonal groups: a 1-brane in eight 9-branes, a 5-brane in four 9-branes, and a 7-brane in two 9-branes. In these cases there are more types of end-solitons than p -branes, which allows the sum of the two charges at the ends to be non-zero. Of course tadpole cancellation requires an excess of exactly 32 9-branes, so these states are not really stable in type I string theory. One could break the $SO(32)$ symmetry with additional Wilson lines and obtain states which would be stable in some energy range. Alternatively one could imagine (via T-duality for example) lower-dimensional orientifold planes, in which these states would be stable.

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Appendix A. Various homotopy groups

The homotopy groups used in this Letter can be found in [11]. The stable homotopy groups of the simple Lie groups are given by

$$\pi_i(U(n)) = \begin{cases} \mathbb{Z} & i \text{ odd} \\ 0 & i \text{ even} \end{cases} \quad (\text{A.1})$$

for $i < 2n$,

$$\pi_i(O(n)) = \begin{cases} \mathbb{Z} & i = 3, 7 \bmod 8 \\ \mathbb{Z}_2 & i = 0, 1 \bmod 8 \\ 0 & i = 2, 4, 5, 6 \bmod 8 \end{cases} \quad (\text{A.2})$$

for $i < n - 1$, and

$$\pi_i(Sp(n)) = \begin{cases} \mathbb{Z} & i = 3, 7 \bmod 8 \\ \mathbb{Z}_2 & i = 4, 5 \bmod 8 \\ 0 & i = 0, 1, 2, 6 \bmod 8 \end{cases} \quad (\text{A.3})$$

for $i < 4n + 2$. Some relevant unstable homotopy groups are

$$\begin{aligned} \pi_1(O(2)) &= \pi_5(O(6)) = \mathbb{Z}, \\ \pi_3(O(4)) &= \pi_7(O(8)) = \mathbb{Z} \oplus \mathbb{Z}. \end{aligned} \quad (\text{A.4})$$

The vacuum manifolds in the Higgs–tachyon model included the Stiefel manifolds

$$V_{n,m} = \frac{G_n}{G_{n-m}}, \quad (\text{A.5})$$

which are real, complex and quaternionic for the cases when G_n is $O(n)$, $U(n)$ and $Sp(n)$, respectively, and the Grassman manifolds

$$M_{n,m} = \frac{G_n}{G_{n-m} \times G_m}. \quad (\text{A.6})$$

The homotopy groups of the Stiefel manifolds $V_{n,m}$ satisfy

$$\pi_i(V_{n,m}) = 0 \quad \text{for } i < \begin{cases} n - m & \mathbb{R}, \\ 2(n - m) + 1 & \mathbb{C}, \\ 4(n - m) + 3 & \mathbb{H}. \end{cases} \quad (\text{A.7})$$

At the upper limit of the above ranges the homotopy groups are given by

$$\pi_{n-m}(V_{n,m}^{\mathbb{R}}) = \begin{cases} \mathbb{Z} & n - m \text{ even}, \\ \mathbb{Z}_2 & n - m \text{ odd}, \end{cases} \quad (\text{A.8})$$

and

$$\pi_{2(n-m)+1}(V_{n,m}^{\mathbb{C}}) = \pi_{4(n-m)+3}(V_{n,m}^{\mathbb{H}}) = \mathbb{Z}. \quad (\text{A.9})$$

There is an exact sequence relating the homotopy groups of G_m , $V_{n,m}$ and $M_{n,m}$:

$$\begin{aligned} \cdots \rightarrow \pi_i(V_{n,m}) \rightarrow \pi_i(M_{n,m}) \rightarrow \pi_{i-1}(G_m) \\ \rightarrow \pi_{i-1}(V_{n,m}) \rightarrow \cdots \end{aligned} \quad (\text{A.10})$$

Using (A.7) it then follows that

$$\pi_i(M_{n,m}) = \pi_{i-1}(G_m) \quad \text{for } i < \begin{cases} n - m & \mathbb{R}, \\ 2(n - m) + 1 & \mathbb{C}, \\ 4(n - m) + 3 & \mathbb{H}. \end{cases} \quad (\text{A.11})$$

For $n - m < m$ a more useful identity is obtained using the equivalence $M_{n,m} = M_{n,n-m}$:

$$\pi_i(M_{n,m}) = \pi_{i-1}(G_{n-m}) \quad \text{for } i < \begin{cases} m & \mathbb{R}, \\ 2m + 1 & \mathbb{C}, \\ 4m + 3 & \mathbb{H}. \end{cases} \quad (\text{A.12})$$

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